Basic Principles of Open-Channel Flows

Table of Contents

Two principles are available for open-channel flows, namely the energy principle and the momentum principle. The former comes from energy conservation and the latter from Newton's second law. For open-channel flows, they are called the specific energy and specific momentum, respectively. Depending on the problem, either the energy or the momentum principle is solved together with the continuity relation.

1. Energy Principle

1.1 Bernoulli Equation and Energy Equation

The Bernoulli equation looks similar to the energy equation. However, these two equations are essentially different, and the ways of applying them differ. The differences between the two equations are summarized below.

The Bernoulli equation can be obtained by integrating the Euler equation(s). The energy equation can be derived from the first law of thermodynamics. We apply the Bernoulli equation to two points, but the energy equation to two channel sections. Therefore, the energy correction factor is introduced to the energy equation due to the non-uniformity of the velocity distribution, but not to the Bernoulli equation. In addition, the energy loss cannot be accounted for in the Bernoulli equation.

The problem is that we cannot apply the energy equation to the open-channel flow. It is because the pressure varies in the depth direction. However, for the pipe flow, since we can assume that the velocity and the pressure are uniform over the cross section, we do not have any such problem.

Table 1. Bernoulli equation versus energy equation*

	Bernoulli equation	Energy equation
source	Euler equation(s)	first law of thermodynamics
application	points	channel sections
energy loss	no	yes
energy correction factor	no	yes
rotational flow	can only be applied along the streamline	no restriction

*The differential form of the energy equation can be obtained by a similar approach to one used for the integral form (Liggett, 1994; Streeter et al., 1998), however, its use is very limited. The energy equation here refers to the integral form derived by the finite control volume approach.

1.2 Energy in Open-channel Flow

With respect to the datum plane, the total head at a section O containing point A is given by

$$H = z_A + d_A \cos\theta + \frac{V_A^2}{2g} \tag{1}$$

where z_A is the elevation above the datum, d_A is the depth measured along the channel section, and $V_A^2/2g$ is the velocity head. Even at the same channel section, every streamline will have a different velocity head because of the non-uniform velocity distribution in actual flow. If the effect of non-uniform velocity is considered, then the total energy at the channel section is

$$H = z + d\cos\theta + \alpha\frac{V^2}{2g} \qquad (2)$$

where α is the energy correction factor. For channels of small slope,

$$H = z + d + \alpha\frac{V^2}{2g} \qquad (3)$$

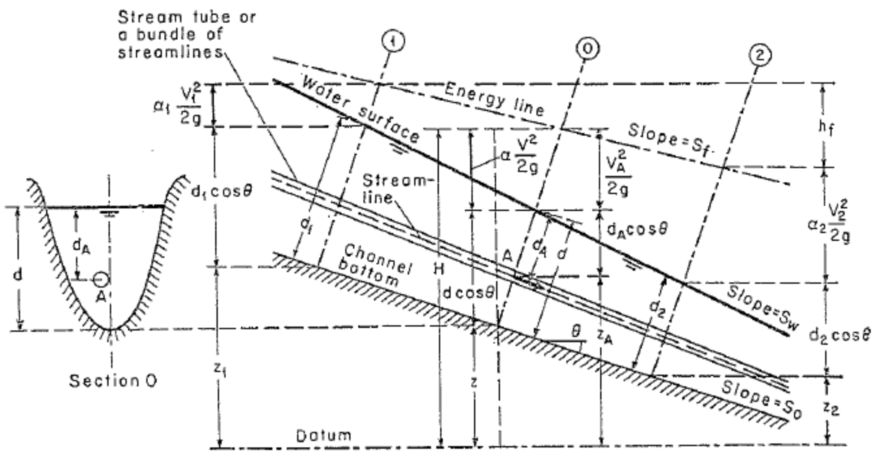

Figure 1. Energy in gradually varied open-channel flow

For a channel of small slope, the conservation of energy at two sections leads to

$$z_1 + d_1 + \alpha_1\frac{V_1^2}{2g} = z_2 + d_2 + \alpha_2\frac{V_2^2}{2g} + h_f \qquad (4)$$

where h_f is the energy loss. Eq.(4) is known as the energy equation. If $\alpha_1 = \alpha_2 = 1$ and $h_f = 0$, then we have

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} = \text{constant} \qquad (5)$$

where y is the flow depth measured vertically upward from the bed, whereas d is the measured downward perpendicularly to the water surface.

1.3 Specific Energy

The specific energy is now defined as the energy at any channel section measured with respect to the channel bottom. That is,

$$E = d\cos\theta + \alpha\frac{V^2}{2g} \qquad (6)$$

For a channel of small slope and α = 1, the specific energy is given by

$$E = y + \frac{V^2}{2g} \qquad (7)$$

which indicates that the specific energy is the sum of the depth of water and the velocity head. It will be shown that the critical flow occurs when the specific energy is minimum.

The specific energy curve shows that, for a given *E*, there are two possible depths, the low stage y_1 and the high stage y_2. The low stage is called the *alternate depth* of the high stage, and vice versa.

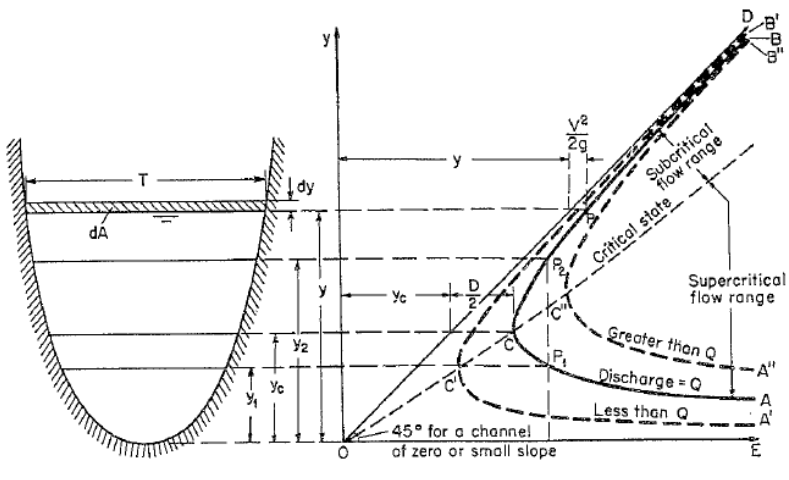

Figure 2. Specific energy curve

1.4 Critical Flow

The critical flow is defined as the condition for which the Froude number is unity. For a constant discharge *Q*, the specific energy is given by

$$E = y + \frac{Q^2}{2gA^2} \tag{8}$$

Differentiating *E* with respect to *y* yields

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 1 - \frac{V^2}{gD} \tag{9}$$

where $dA/dy = T$ is used because $dA = Tdy$ near the free surface. If $dE/dy = 0$, then

$$\frac{V^2}{2g} = \frac{D}{2} \tag{10}$$

which indicates that the velocity head is half the hydraulic depth for the critical flow. Eq.(10) is equivalent to

$$Fr = \frac{V}{\sqrt{gD}} = 1 \tag{11}$$

For a channel of large slope, whose energy correction factor cannot be assume to be unity, the Froude number is given by

$$Fr = \frac{V}{\sqrt{gD\cos\theta/\alpha}} \tag{12}$$

which obviously comes from

$$\alpha\frac{V^2}{2g} = \frac{D\cos\theta}{2}$$

1.5 Interpretation of Local Phenomena

Let the flow depth before and after the hydraulic jump y_1 and y_2. These depths are called the initial depth and *sequent depth* (or *conjugate depth*), respectively, and are shown on the specific force curve. They should be distinguished from the *alternate depths* y_1 and $y_2^{'}$, which are two possible depths for the same specific energy. Figure 3 clearly shows that alternate depths occur at the same specific energy and the initial and sequent depths occur at the same specific force.

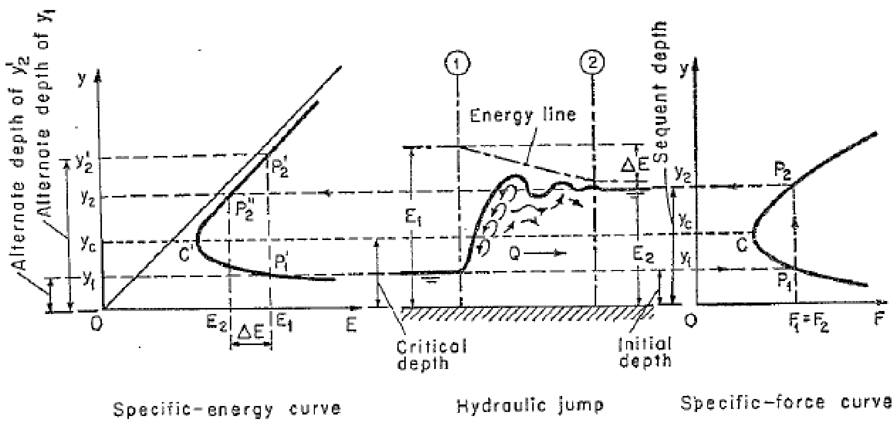

Figure 3. Specific energy and specific force curves for the hydraulic jump

2. Momentum Principle

2.1 Momentum in Open-channel Flow

According to Newton's second law of motion, the change of momentum per unit of time in the body of water in a flowing channel is the resultant of all external forces that are acting on the body. If we apply Newton's second law to a channel of large slope, we have

$$\rho Q\left(V_2 - V_1\right) = P_1 - P_2 + W\sin\theta - F_f \tag{13}$$

where F_f is the friction force on the channel bed.

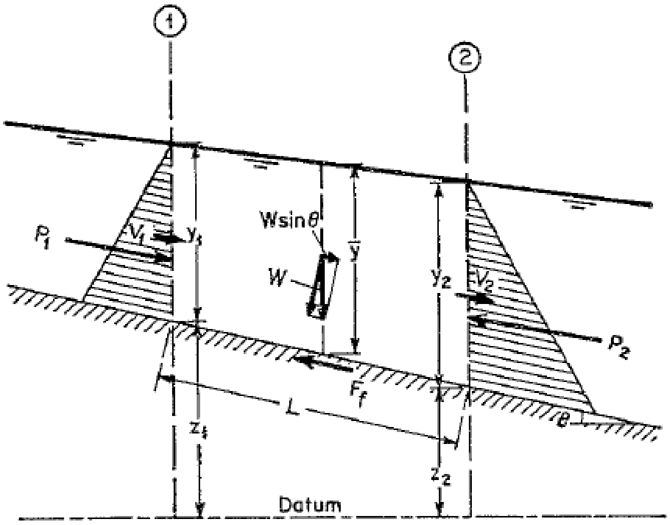

Figure 4. Application of momentum principle

For a parallel flow or a gradually varied flow, the pressure force P can be computed by assuming the hydrostatic pressure distribution. However, for a curvilinear flow or rapidly varied flow, the pressure must be corrected for the curvature effect of the streamlines. For simplicity, P can be replaced with $\beta' P$, where the pressure coefficient β' is defined by

$$\beta' = \frac{1}{A\bar{z}}\int_A h dA = 1 + \frac{1}{A\bar{z}}\int_A c dA \qquad (14)$$

where $\bar{z}$ is the depth to the centroid of water area A and c is the pressure head correction. The pressure coefficient is unity for the parallel flow, but is greater than and less than unity for the concave and convex flows, respectively.

For a short reach of a rectangular channel of small slope and width b, we have

$$P_1 = \frac{1}{2} w b y_1^2$$

$$P_2 = \frac{1}{2} w b y_2^2$$

$$Q = \frac{1}{2}\left(V_1 + V_2\right) b \overline{y}$$

$$W = w b \overline{y} L$$

where $\overline{y}$ is the average depth (=$1/2\left(y_1 + y_2\right)$). For the bed friction, assume

$$F_f = w h_f^{'} b \overline{y} \tag{15}$$

where $h_f^{'}$ is the friction head. Substituting the above expressions in Eq.(13) leads to

$$z_1 + y_1 + \beta_1 \frac{V_1^2}{2g} = z_2 + y_2 + \beta_2 \frac{V_2^2}{2g} + h_f^{'} \tag{16}$$

which is very similar to the energy equation.

In the energy equation, h_f measures the internal energy dissipated in the whole mass of the water in the reach, while $h_f^{'}$ in the momentum equation measures losses due to external forces exerted on the water by the walls of the channel (Chow, 1959).

2.2 Specific Force

For a short horizontal reach of a prismatic channel, Eq.(13) becomes

$$\rho Q\left(V_2 - V_1\right) = P_1 - P_2 \tag{17}$$

where $P_1 = w\overline{z_1} A_1$ and $P_2 = w\overline{z_2} A_2$. The momentum correction factor β is assumed to be unity. Therefore, we have

$$\frac{Q^2}{gA_1} + \overline{z_1} A_1 = \frac{Q^2}{gA_2} + \overline{z_2} A_2 \tag{18}$$

Then, we define the specific force as

$$F = \frac{Q^2}{gA} + \bar{z} A \tag{19}$$

where the first term is the momentum of the flow per unit time and per unit weight of water, and the second term is the force per unit weight of water.

If we differentiate Eq.(19) with respect to y, we have

$$\frac{dF}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{d}{dy}\left(\bar{z}A\right) = 0$$

which yields

$$\frac{V^2}{2g} = \frac{D}{2} \tag{20}$$

This indicates that the specific force is minimum for a given discharge at the critical state of flow.

3. Critical Flow and Control Section

The critical flow is defined by the flow whose Froude number is unity. Under the critical flow condition, both the specific energy and specific force is minimum for the given discharge.

The control section is defined by the section where the discharge can be calculated once the depth is known. Hydraulic structures that cause the critical flow are used as a control section for open-channel flows. Examples of control sections include spillway crest, weir, gate, overfall, etc. These structures provide a means to record the flow rates simply by measuring the critical depths.

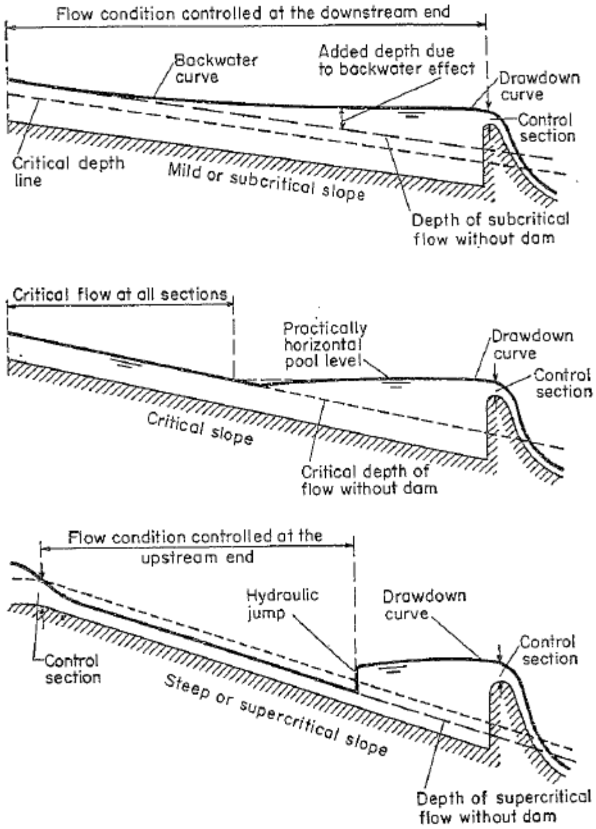

Figure 5. Control sections for subcritical, critical, and supercritical flows.

References

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Streeter, V.L., Wylie, E.B., and Bedford, K.W. (1998). *Fluid Mechanics* (9th ed.), McGraw-Hill, New York, NY.